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Unsteady mixed convection boundary layer flow near the stagnation point on a vertical surface in a porous medium

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Abstract

The unsteady mixed convection boundary layer flow near the region of a stagnation point on a vertical surface embedded in a Darcian fluid-saturated porous medium is studied in this paper. It is assumed that the unsteadiness is caused by the impulsive motion of the free stream velocity and by sudden increase in the surface temperature. The problem is reduced to a single partial differential equation, which is solved numerically using the Keller–Box method. The small time (initial unsteady flow) as well as the large time (final steady state flow) solutions are also included in the analysis. The asymptotic behavior of the solution for small and large values of the mixed convection parameter λ is also examined when the flow becomes steady. It is shown that there is a smooth transition from the small time solution to the large time solution. It is also shown that there is an excellent agreement between the numerical and analytical solutions. The uniqueness of this problem lies on the fact that we have been able to show that in the case of steady state flow, solutions are possible for all values of $\lambda > 0$ (assisting flow) and for $\lambda < 0$ (opposing flow), solutions are possible only for a limited range of λ.

Keywords: Unsteady flow; Mixed convection; Boundary layer; Flat plate; Stagnation point; Analytical solution; Numerical solution

1. Introduction

Convective heat transfer in fluid-saturated porous media has received much attention in recent years because of its important applications both in technology and geothermal energy recovery. These applications include oil recovery, food processing, fiber and granular insulation, design of packed bed reactors, the dispersion of chemical contaminants in various processes in the chemical industry and in the environment, to name but a few. Numerous authors cite a wide variety of these applications involving convective transport phenomena. Several others investigate the intricate nature of solution structure from a fundamental point of view in idealizing settings. Detailed reviews of the subject including exhaustive lists of references, were recently performed by Ingham and Pop [1], Nield and Bejan [2], Vafai [3], Pop and Ingham [4], and Bejan and Kraus [5].

Most of the recent research on convective flow in porous media has been directed to the problems of steady free and mixed convection flows over heated bodies embedded in fluid-saturated porous media. However, unsteady convective boundary layer flow problems have not, so far, received as much attention. Perhaps, the first study on unsteady boundary layer flow on flat surfaces in porous media was made by Johnson and Cheng [6] who found similarity solutions for certain variations of the wall temperature. The more common cases, in general, involve transient convection, which is non-similar and

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Nomenclature

C_f	skin friction coefficient
f	reduced stream function
g	acceleration due to gravity
K	permeability of the porous medium
L	characteristic length
Nu	Nusselt number
Pe	Péclet number
Pr	Prandtl number
Ra	Rayleigh number
Re	Reynolds number
t	time
T	fluid temperature
u, v	velocity components along x- and y-axes, respectively
U_e	characteristic velocity
x, y	Cartesian coordinates along and normal to the surface, respectively

Greek symbols

α_m	effective thermal diffusivity
β	coefficient of thermal expansion
η	pseudo-similarity variable
θ	non-dimensional fluid temperature
τ	non-dimensional time
λ	mixed convection parameter
ν	kinematic viscosity
ξ	non-dimensional transformed variable
σ	ratio of composite material heat capacity to convective fluid heat capacity

Subscripts

e, w, ∞	conditions at the edge of the boundary layer, at the surface, and in the free stream, respectively

Superscript

$'$	differentiation with respect to η or $\bar{\eta}$

hence, more complicated mathematically. The interested reader can find an excellent collection of papers on unsteady convective flow problems over heated bodies embedded in a fluid-saturated porous medium in the review papers by Pop et al. [7], Bradean et al. [8] and in the book by Pop and Ingham [4].

Motivation to study mixed convection in porous media comes from the need to characterize the convective transport processes around deep geological repository for the disposal of high-level nuclear waste, e.g. spent fuel rods from nuclear reactors (see [9]). Presently proposed repositories would cover an area of up to 5 km^2 and 600 m below ground level. The parameters which are expected to affect the temperature field around a repository include the natural stratigraphy of the site, groundwater flow caused by the hydrostatic head of the water table, layout of tunnel and rooms, and the variation in waste heat generation with time (see [9]).

Cheng [10] seems to be the first to consider the problem of steady mixed convection in porous media along inclined surfaces. Both aiding (assisting) and opposing flows were considered. Assuming a power law variation of the wall temperature ($T_w = T_\infty \pm Ax^m$, where A and m are constants), similarity solutions were obtained for two cases: (i) a uniform flow along a vertical isothermal flat plate ($m = 0$); and (ii) an accelerating flow over a 45° inclined flat plate of constant heat flux ($m = 1/3$). The heat transfer rate is found to approach asymptotically the forced and free convection limits as the value of the governing mixed convection parameter Ra/Pe approaches zero and infinity, respectively. Merkin [11] has studied later the steady mixed convection boundary layer flow adjacent to a vertical, uniform heat flux flat plate embedded in a porous medium. As pointed out by Cheng [10], a similarity solution does not exist in this case so that Merkin [11] used two different coordinates perturbations for small and large downstream distances. These expansions were then matched for intermediate downstream distances using numerical integration of the governing equations. Joshi and Gebhart [12] extended Merkin's analysis by using the method of matched asymptotic expansions. It was shown that the first correction to the boundary layer theory (neglected by Merkin [11]) occurs at the same level as the second correction due to mixed convection. Therefore, both of these effects must be included in a physically consistent analysis.

A review of the literature shows that very little research has been reported on unsteady mixed convection flow in porous media. Harris et al. [13] have performed an analysis of the unsteady mixed convection boundary layer flow from a vertical flat plate embedded in a porous medium. A complete analysis was made at the initial unsteady flow ($t = 0$) and the steady state flow at large times ($t \to \infty$), and a series solution valid at small times obtained using semi-similar coordinates originated by Smith [14].

The purpose of this paper is to study the unsteady mixed convection flow near the stagnation point on a heated vertical flat plate embedded in a fluid-saturated porous medium, in the presence of buoyancy forces. It is assumed that the unsteadiness is caused by the impulsive start in motion of the free stream flow and by the sudden increase or sudden decrease in the surface temperature, which is considered to vary linearly with the distance along the plate. The governing Darcy and energy equations are transformed using self-similar coordinates found by Williams and Rhyne [15] for the case of the boundary layer development on a wedge impulsively set into motion in a viscous and incompressible fluid. This is the method of semi-similar solutions, in which the number of independent variables is reduced from three to two by an appropriate scaling. The scale of time has been selected in such a manner that the traditional infinite region is transformed to a finite region, which reduces the computational time considered. Thus, the transformed Darcy and energy boundary layer equations are solved numerically for the whole transient regime using the Keller–Box method described by Cebeci and Bradshaw [16], when the buoyancy parameter λ is positive (assisting flow) and negative (opposing flow). Also, a closed form solution of these equations has been shown to exist at the time $\tau = 0$ (initial unsteady flow), $\tau \to \infty$ (final steady state flow) and for small times τ (non-dimensional). Particular cases of the present results are compared with those of Merkin [11] and Crane [17]. The results are believed to be important to future theoretical studies of convective flow problems in porous media. It is of some importance to mention at this place that mixed convection in stagnation flows becomes important when the buoyancy forces, due to the temperature difference between the wall and the free stream, become high and thereby modify the flow and thermal fields significantly. In such situations, the flow and thermal fields are no longer symmetric with respect to the stagnation line [18].

2. Basic equations

Let us suppose that this investigation is appropriate to the mixed convection flow at the two-dimensional stagnation point on a double-infinite vertical flat plate, which is embedded in a fluid-saturated porous medium of constant ambient temperature T_∞. It is assumed that at time $t = 0$, the external flow starts impulsively in motion from rest towards the plate with a steady velocity $u_e(x)$. The flow configuration is shown schematically in Fig. 1 together with the corresponding Cartesian coordinates (x, y) in vertical and horizontal directions, respectively, with the positive y-axis pointing towards the porous medium (external flow). Either heating or cooling of the plate is assumed to begin simultaneously with the motion of the external stream. In particular, it is assumed that the temperature of the plate $T_w(x)$ varies linearly with the distance x along the

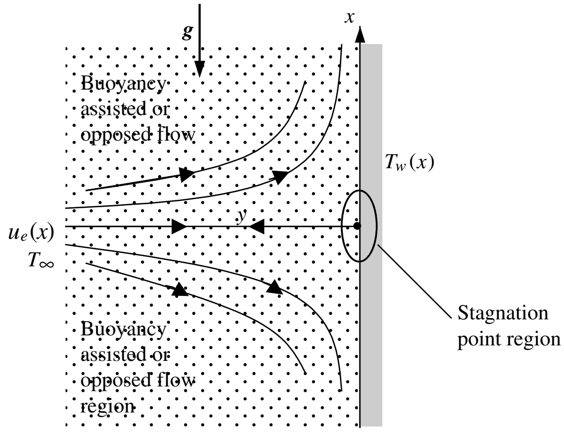

Fig. 1. Physical model and coordinate system.

plate. Therefore, the mixed convection flow is symmetric about the centreline plane, which contains the stagnation point. Thus, the plate temperature and the condition far from the plate is assumed to be given by

$$T_w(x) = T_\infty + sT_0(x/L), \quad u_e(x) = U_e(x/L) \tag{1}$$

where L is a characteristic length, $T_0 > 0$ is a characteristic temperature and $s = \pm 1$. The case $s = +1$ corresponds to the situation when $T_w(x)$ increases (linearly) from $x = -\infty$ to $x = +\infty$, while $s = -1$ corresponds to the case when $T_w(x)$ decreases (linearly) from $x = +\infty$ to $x = -\infty$, respectively. In the first case ($s = +1$), the buoyancy force accelerates the motion in both, upper half plane (up flow) and lower half plane (down flow). Both of these flows are assisting flows and are equivalent at $x = +\infty$. On the other hand, in the second case ($s = -1$), the buoyancy force decelerates the flow both in the upper half plane and in the lower half plane. Both of these flows are opposing flows and are equivalent at $x = -\infty$. Since the present flow model is symmetric, it is sufficed to consider the flow only in the upper half plane for $s = \pm 1$. It is assumed that the convective fluid and the porous medium are everywhere in the local thermodynamic equilibrium, the temperature of the fluid is everywhere below the boiling point and that the properties of the fluid and the porous medium such as viscosity, thermal conductivity, thermal expansion coefficient, specific heat and permeability are constants. Under these assumptions along with the Darcy–Boussinesq approximation, the unsteady boundary layer equations governing this mixed convection flow are given by (see [4])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$u = u_e(x) + \frac{gK\beta}{\nu}(T - T_\infty) \tag{3}$$

$$\sigma\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2} \tag{4}$$

subject to the initial and boundary conditions

$$\begin{aligned}&t<0: \quad u(x,y)=v(x,y)=0, \quad T(x,y)=T_\infty \quad \text{any } x,y\\ &t\geqslant 0: \quad v(x,0)=0, \quad T(x,0)=T_w(x)-T_\infty=sT_0(x/L),\\ &\quad x\geqslant 0\\ &u(x,\infty)=u_e(x)=(U_e/L)x, \quad x\geqslant 0\end{aligned} \tag{5}$$

where u and v are the velocity components along x- and y-axes, T is the fluid temperature, g is the gravitational acceleration, K is the permeability of the porous medium, α_m is the effective thermal diffusivity of the porous medium, β is thermal expansion coefficient, v is the kinematic viscosity and σ is the ratio of composite material heat capacity to convective fluid heat capacity.

Following Williams and Rhyne [15], and Seshadri et al. [19], we introduce the following new variables

$$\begin{aligned}&\eta=(U_e/L\alpha_m)^{1/2}y\xi^{-1/2}, \quad \xi=1-\exp(-\tau),\\ &\quad \tau=(U_e/L\sigma)t\\ &u(x,y,t)=(U_e x/L)f'(\xi,\eta), \quad v=-(U_e\alpha_m/L)^{1/2}\xi^{1/2}f(\xi,\eta)\\ &T(x,y,t)=T_\infty+sT_0(x/L)\theta(\xi,\eta)\end{aligned} \tag{6}$$

for $0\leqslant\xi\leqslant 1$. Using these transformations, Eqs. (2)–(4) become

$$f'=1+\lambda\theta \tag{7}$$

$$\theta''+\frac{1}{2}\eta(1-\xi)\theta'+\xi(f\theta'-f'\theta)=\xi(1-\xi)\frac{\partial\theta}{\partial\xi} \tag{8}$$

for $0\leqslant\xi\leqslant 1$. The boundary conditions (5) now become

$$f(\xi,0)=0, \quad \theta(\xi,0)=1, \quad \theta(\xi,\eta)\to 0 \quad \text{as } \eta\to\infty \tag{9}$$

for $0\leqslant\xi\leqslant 1$. Here λ (= constant) is the mixed convection parameter and is defined as

$$\lambda=s\frac{Ra}{Pe}, \quad Ra=\frac{gK\beta T_0 L}{\alpha_m v}, \quad Pe=\frac{U_e L}{\alpha_m} \tag{10}$$

with Ra and Pe being the Rayleigh and Péclet numbers, respectively. It should be noted that $\lambda>0$ ($s=+1$) corresponds to buoyancy assisting flow and $\lambda<0$ ($s=-1$) corresponds to buoyancy opposing flow. Thus, Eqs. (7) and (8) can be combined to give the following equation:

$$f'''-\frac{1}{2}\eta(1-\xi)f''+\xi(ff''+f'-f'^2)=\xi(1-\xi)\frac{\partial f'}{\partial\xi} \tag{11}$$

for $0\leqslant\xi\leqslant 1$, and is subject to the boundary conditions

$$f(\xi,0)=0, \quad f'(\xi,0)=1+\lambda, \quad f'(\xi,\infty)=1 \tag{12}$$

for $0\leqslant\xi\leqslant 1$. We notice that Eq. (11) subjected to the boundary conditions (12) is a non-linear parabolic partial differential equation, but for $\xi=0$ ($\tau=0$) and $\xi=1$ ($\tau\to\infty$), it reduces to ordinary differential equations. The physical parameters of interest are the skin friction coefficient C_f, and the Nusselt number Nu, which are defined as

$$C_f=\frac{2\mu(x/L)}{\rho u_e^2(x)}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad Nu=\frac{L}{(T_w-T_\infty)}\left(-\frac{\partial T}{\partial y}\right)_{y=0} \tag{13}$$

Using variables (6), we get

$$\begin{aligned}&C_f/(Pr/Re)^{1/2}=2\xi^{-1/2}f''(\xi,0),\\ &Nu/(Pr\,Re)^{1/2}=\xi^{-1/2}[-\theta'(\xi,0)]\end{aligned} \tag{14}$$

or

$$Nu/(Pr\,Re)^{1/2}=\frac{1}{\lambda}\xi^{-1/2}[-f''(\xi,0)] \tag{15}$$

where $Pr=v/\alpha_m$ and $Re=U_eL/v$ are the Prandtl and Reynolds numbers, respectively.

3. Solution

The governing partial differential equation (11), and the associated boundary conditions (12), permit separate reductions to ordinary differential systems governing the profiles of the non-dimensional velocity and temperature functions in the initial unsteady state flow at $\xi=0$, final steady state flow at large times given by $\xi=1$ and for small times ξ or τ.

Initial unsteady flow. This solution corresponds to $\xi=0$ ($\tau=0$), where $f(0,\eta)=F(\eta)$. In this case, Eq. (11) can be reduced to

$$F'''+\frac{1}{2}\eta F''=0 \tag{16}$$

subject to the boundary conditions

$$F(0)=0, \quad F'(0)=1+\lambda, \quad F'(0)=1 \tag{17}$$

The analytical solution of these equations is given by

$$\begin{aligned}&F=\eta+\lambda\left[\eta\,\mathrm{erfc}(\eta/2)-\frac{2}{\sqrt{\pi}}\exp(-\eta^2/4)+\frac{2}{\sqrt{\pi}}\right]\\ &F'=1+\lambda\,\mathrm{erfc}(\eta/2)\end{aligned} \tag{18}$$

where $\mathrm{erfc}(z)$ is the complementary error function which is defined as

$$\mathrm{erfc}(z)=\frac{2}{\sqrt{\pi}}\int_\eta^\infty e^{-z^2}\,dz \tag{19}$$

Solution (18) can then be used to calculate the local Nusselt number in the initial unsteady state flow, namely

$$Nu/(Pr\,Re)^{1/2} = \frac{1}{\sqrt{\pi}}\xi^{-1/2} \tag{20}$$

Final steady state flow. This solution corresponds to $\xi = 1$ ($\tau \to \infty$), where $f(1, \eta) = G(\eta)$. Eq. (11) now becomes

$$G''' + GG'' + G' - G'^2 = 0 \tag{21}$$

subject to the boundary conditions

$$G(0) = 0, \quad G'(0) = 1 + \lambda, \quad G'(\infty) = 1 \tag{22}$$

(i) *Solution for small* λ. It is possible to obtain an approximate solution of Eqs. (21) and (22) for small values of λ. In this case, we seek a power series solution of these equations of the form

$$G(\eta) = G_0(\eta) + G_1(\eta)\lambda + G_2(\eta)\lambda^2 + \text{h.o.t.} \tag{23}$$

which is valid for $|\lambda| \ll 1$, where the functions G_0, G_1 and G_2 are given by the following three sets of equations

$$\begin{aligned} &G_0''' + G_0G_0'' + G_0' - G_0'^2 = 0 \\ &G_0(0) = 0, \quad G_0'(0) = 1, \quad G_0'(\infty) = 1; \end{aligned} \tag{24}$$

$$\begin{aligned} &G_1''' + G_0G_1'' + G_1' - 2G_0'G_1' = 0 \\ &G_0(0) = 0, \quad G_1'(0) = 1, \quad G_1'(\infty) = 0; \end{aligned} \tag{25}$$

$$\begin{aligned} &G_2''' + G_0G_2'' + G_2' - 2G_0'G_2' = G_1'^2 - G_1G_1'' \\ &G_2(0) = G_2'(0) = 0, \quad G_2'(\infty) = 0 \end{aligned} \tag{26}$$

Eqs. (24)–(26) have the analytical solutions

$$\begin{aligned} G_0 &= \eta \\ G_1' &= -\eta\sqrt{\frac{\pi}{2}}\operatorname{erfc}(\eta/\sqrt{2}) + e^{-\eta^2/2} \\ G_2' &= \sqrt{\pi}\left(\frac{\pi}{2} - \frac{7}{3}\right)\left[\frac{\eta}{\sqrt{2}}\operatorname{erfc}(\eta/\sqrt{2}) - \frac{1}{\sqrt{\pi}}e^{-\eta^2/2}\right] \\ &\quad + \frac{\pi}{4}(3 + \eta^2)\operatorname{erfc}^2(\eta/\sqrt{2}) \\ &\quad - \frac{5}{6}\eta e^{-\eta^2/2}\operatorname{erfc}(\eta/\sqrt{2}) - \frac{\pi}{4}\operatorname{erfc}(\eta/\sqrt{2}) \\ &\quad - \frac{7}{3}e^{-\eta^2} + \frac{8}{3}\sqrt{\pi}\eta\operatorname{erfc}(\eta) \end{aligned} \tag{27}$$

Thus, we have

$$\begin{aligned} &G_0''(0) = 0, \quad G_1''(0) = -\sqrt{\frac{\pi}{2}} = -1.2533, \\ &G_2''(0) = \sqrt{\pi}\left(\frac{\sqrt{2}}{4}\pi + \frac{8}{3} - \frac{17\sqrt{2}}{6}\right) = -0.4068 \end{aligned} \tag{28}$$

and

$$G''(0) = -1.2533\lambda - 0.4068\lambda^2 + \text{h.o.t.} \tag{29}$$

for $|\lambda| \ll 1$.

(ii) *Solution for large* λ. In this case, we take

$$G(\eta) = \lambda^{1/2}\overline{G}(\bar{\eta}), \quad \bar{\eta} = \lambda^{1/2}\eta \tag{30}$$

so that Eq. (21) becomes

$$\overline{G}''' + \overline{GG}'' - \overline{G}'^2 + \lambda^{-1}\overline{G}' = 0 \tag{31}$$

subject to

$$\overline{G}(0) = 0, \quad \overline{G}'(0) = 1 + \lambda^{-1}, \quad \overline{G}'(\infty) = \lambda^{-1} \tag{32}$$

where primes now denote differentiation with respect to $\bar{\eta}$. We look for a solution of Eqs. (31) and (32) of the form

$$\overline{G}(\bar{\eta}) = \overline{G}_0(\bar{\eta}) + \overline{G}_1(\bar{\eta})\lambda^{-1} + \overline{G}_2(\bar{\eta})\lambda^{-2} + \text{h.o.t.} \tag{33}$$

which is valid for $|\lambda| \gg 1$. The solution for $\overline{G}_0$ is given by equation

$$\overline{G}_0''' + \overline{G}_0\overline{G}_0'' - \overline{G}_0'^2 = 0 \tag{34}$$

subject to the boundary conditions

$$\overline{G}_0(0) = 0, \quad \overline{G}_0'(0) = 1, \quad \overline{G}_0'(\infty) = 0 \tag{35}$$

These equations describe the flow due to a stretching wall, first studied by Crane [17], and have the solution

$$\overline{G}_0 = 1 - e^{-\bar{\eta}} \tag{36}$$

The functions $\overline{G_1}$ and $\overline{G}_2$ are determined numerically from the equations

$$\begin{aligned} &\overline{G}_1''' + \overline{G}_1'' - e^{-\bar{\eta}}\left(\overline{G}_1'' + 2\overline{G}_1' + \overline{G}_1 - 1\right) = 0 \\ &\overline{G}_1(0), \quad \overline{G}_1'(0) = 1, \quad \overline{G}_1'(\infty) = 1; \end{aligned} \tag{37}$$

$$\begin{aligned} &\overline{G}_2''' + \overline{G}_2'' - e^{-\bar{\eta}}\left(\overline{G}_2'' + 2\overline{G}_2' + \overline{G}_2\right) = \overline{G}_1'^2 - \overline{G}_1\overline{G}_1'' - \overline{G}_1' \\ &\overline{G}_2(0) = \overline{G}_2'(0) = 0, \quad \overline{G}_2'(\infty) = 0 \end{aligned} \tag{38}$$

Thus, we have

$$\begin{aligned} G''(0) &= \lambda^{3/2}\left(\overline{G}_0''(0) + \overline{G}_1''(0)\lambda^{-1} + \overline{G}_2''(0)\lambda^{-2} + \text{h.o.t.}\right) \\ &= \lambda^{3/2}\left(-1 - 0.8160\lambda^{-1} + 0.4605\lambda^{-2} + \text{h.o.t.}\right) \end{aligned} \tag{39}$$

for $|\lambda| \gg 1$.

Solution for small ξ *and* τ. We assume now that a solution of Eq. (11) for small values of ξ ($\ll 1$) is of the form

$$f = f_0(\eta) + f_1(\eta)\xi + f_2(\eta)\xi^2 + \text{h.o.t.} \tag{40}$$

where f_0 is given by expression (18) and the functions f_1 and f_2 are determined from the equations

$$f_1''' + \frac{1}{2}\eta f_1'' - f_1' = \frac{1}{2}\eta f_0'' - f_0 f_0'' - f_0' + f_0'^2$$
$$f_1(0) = f_1'(0) = 0, \quad f_1'(\infty) = 0; \tag{41}$$

$$f_2''' + \frac{1}{2}\eta f_2'' - f_2' = \frac{1}{2}\eta f_1'' + 2f_0'f_1' - f_1 f_0'' - f_0 f_1'' - 2f_1'$$
$$f_2(0) = f_2'(0) = 0, \quad f_2'(\infty) = 0 \tag{42}$$

The solution of Eq. (41) is

$$f_1' = \lambda\left(1 + \frac{\lambda}{2} - \frac{2\lambda}{3\pi}\right)\left[\left(1 + \frac{1}{2}\eta^2\right)\mathrm{erfc}(\eta/2) - \frac{1}{\sqrt{\pi}}\eta e^{-\eta^2/4}\right] + \frac{\lambda^2}{2}\left(-1 + \frac{1}{2}\eta^2\right)\mathrm{erfc}^2(\eta/2) - \lambda\left(1 + \frac{3\lambda}{2\sqrt{\pi}}\eta e^{-\eta^2/4}\right)\mathrm{erfc}(\eta/2) + \frac{2}{\pi}\lambda^2 e^{-\eta^2/2} - \frac{\lambda}{4\sqrt{\pi}}\eta e^{-\eta^2/4} - \frac{4\lambda^2}{3\pi}e^{-\eta^2/4} \tag{43}$$

with

$$f_1''(0) = \frac{\lambda}{\sqrt{\pi}}\left(\frac{4\lambda}{3\pi} - \frac{5 + 6\lambda}{4}\right) \tag{44}$$

while Eq. (42) can be easily solved numerically.

Thus, the local Nusselt number given by expression (15) becomes

$$Nu/(Pr\,Re)^{1/2} = \frac{1}{\lambda}\xi^{-1/2}[-f''(\xi,0)] = -\frac{1}{\lambda}\xi^{-1/2}\left[f_0''(0) + f_1''(0)\xi + f_2''(0)\xi^2 + \text{h.o.t.}\right] = \frac{1}{\sqrt{\pi}}\left[\xi^{-1/2} - \left(\frac{4\lambda}{3\pi} - \frac{5 + 6\lambda}{4}\right)\xi^{1/2} + \text{h.o.t.}\right] \tag{45}$$

for ξ and τ small ($\ll 1$), where

$$\xi = \tau - \frac{1}{2}\tau^2 + \frac{1}{6}\tau^3 + \text{h.o.t.} \tag{46}$$

4. Results and discussion

In order to determine the evolution of the boundary layer, the governing partial differential equation (11) subject to the boundary conditions (12) has been solved numerically using the Keller–Box method, as described by Cebeci and Bradshaw [16]. The computation starts with the initial analytical solution (18) at $\xi = 0$ corresponding to $\tau = 0$ and proceed up to the steady state solution at $\xi = 1$ corresponding to $\tau \to \infty$. The solution of the ordinary differential equations (21) and (22), which govern the solution behaviour for the final steady state flow at $\xi = 1$ for all values of λ, and the solution of Eqs. (37) and (38), which govern the solution behaviour for the final steady state flow when $\lambda \gg 1$, have been obtained using the Keller–Box method. It is found that for $\lambda > 0$ (assisting flow), solutions of Eqs. (21) and (22) can be obtained for all λ, while for $\lambda < 0$ (opposing flow) these equations have solutions only in the range of $\lambda \geqslant \lambda_0 = -1.4175$. However, for λ in the range of $\lambda_0 < \lambda < -1$, the solution is not unique, there being two solutions $G_1(\eta)$ and $G_2(\eta)$ for a given λ. One solution continuing from the lower stagnation point solution, and the other is such that $G''(0) \to G^*$ (<0) as $\lambda \to -1$, where the exact value of G^* cannot be determined. This can be seen from Fig. 2, where $G''(0)$ is plotted as a function of λ. The values of $G_1''(0)$ and $G_2''(0)$ are given in Table 1 for λ in the range of $-1.417 \leqslant \lambda \leqslant -1$. It is seen that these dual solutions have $G_2''(0) \leqslant G_1''(0)$ for a given value of λ in the range of $-1.417 \leqslant \lambda \leqslant -1$. It is worth mentioning that such dual solutions were first observed in porous media by Merkin [11] for the problem of mixed convection boundary layer flow on a vertical surface embedded in a fluid-saturated porous medium. Fig. 2 also demonstrates that the expression (29) provides a very good approximation to the heat transfer for small values of λ ($\ll 1$). Further, Fig. 3 illustrates the behaviour of the numerical solution $G''(0)$ of Eq. (21) at large values of λ ($\gg 1$). The asymptotic solutions, given by Eqs. (29) and (39) are also included in this figure. It is seen that the function $G''(0)$ given by (39) provides an asymptote to which the wall heat transfer tends to as $\lambda \to \infty$.

Fig. 4 illustrates the variation of the initial unsteady velocity profiles $F'(\eta)$ at $\xi = 0$ for some values of λ. It is seen that we have, (i) for λ in the range of $-1 \leqslant \lambda < 0$, the fluid velocity reduces as we approach the stagnation point region and is lower than the free stream flow; (ii) at

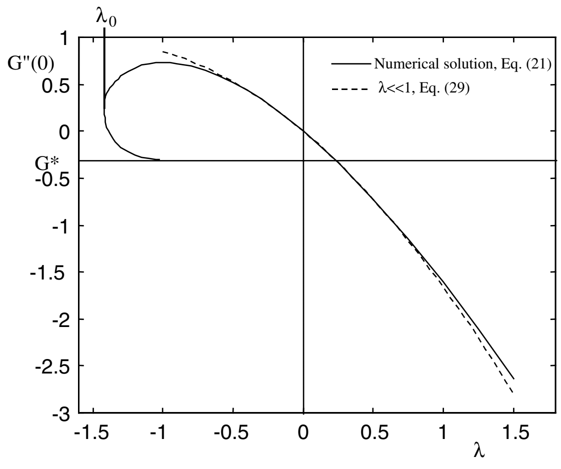

Fig. 2. Variation of the reduced skin friction $G''(0)$ with λ (small) for the final steady state flow ($\xi = 1$).

Table 1
Values of $G''(0)$ for $-1.4175 < \lambda \leqslant -1$

λ	$G_1''(0)$	$G_2''(0)$
−1.00	0.7315	
−1.05	0.7286	−0.3007
−1.10	0.7190	−0.2933
−1.15	0.7019	−0.2791
−1.20	0.6757	−0.2561
−1.25	0.6381	−0.2216
−1.30	0.5858	−0.1711
−1.32	0.5585	−0.1446
−1.34	0.5302	−0.1131
−1.36	0.4930	−0.0747
−1.38	0.4580	−0.0259
−1.3885	0.4367	0.0000
−1.40	0.4040	0.0431
−1.41	0.3701	0.0966
−1.417	0.3420	0.1733

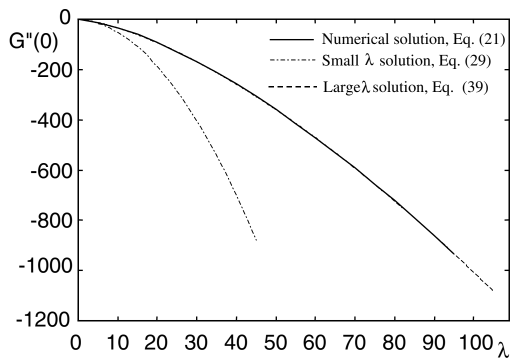

Fig. 3. Variation of the reduced skin friction $G''(0)$ with λ (large) for the final steady state flow ($\xi = 1$).

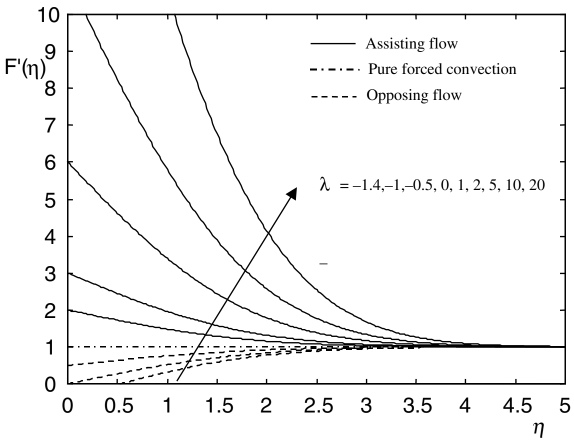

Fig. 4. Profiles of the non-dimensional velocity $F'(\eta)$ for the initial unsteady state flow ($\xi = 0$), for $\lambda \geqslant -1$.

$\lambda = 0$, pure forced convection; (iii) for $\lambda > 0$, the boundary layer thickness reduces as λ increases since within the boundary layer, the fluid velocity is becoming increasingly greater than the free stream flow; (iv) as $\lambda \to \infty$ pure free convection.

The variations of the local Nusselt number with ξ are shown in Fig. 5, for a range of values of λ. The final steady state solution ($\xi = 1$) given by Eqs. (21) and (22) is also included here. It is also shown that the transition from initial unsteady flow to the final steady flow takes place smoothly. As λ increases, the local Nusselt number also increases.

Finally, Fig. 6 shows the variation of the local Nusselt number with τ obtained by solving numerically Eqs. (11) and (12) for different values of λ. The small time analytical solution as given by the series (45) is also included in this figure. We can see that the agreement between the numerical and analytical small τ solution is very good for small values of λ, while for large values of λ this agreement is not good enough. However, the

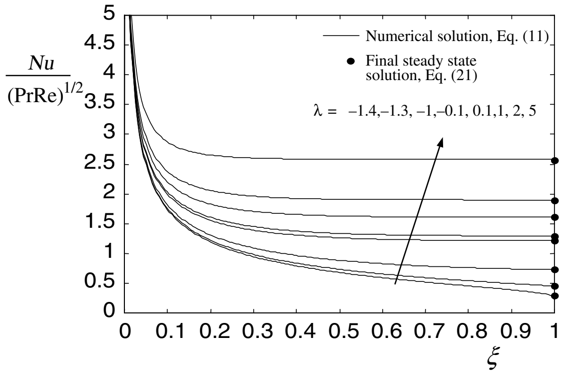

Fig. 5. Variation of the local Nusselt number with ξ for some values of λ.

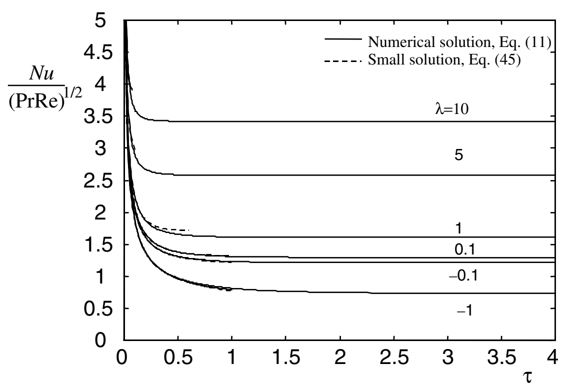

Fig. 6. Variation of the local Nusselt number with time τ for $\lambda \geqslant -1$.

agreement between these two solutions can be improved if more terms in the series (45) are considered.

5. Conclusions

The problem of unsteady mixed convection boundary layer flow near the region of a stagnation point on a heated vertical surface embedded in a fluid-saturated porous medium is studied in this paper. This problem occurs as the surface temperature of the plate, which is assumed to vary linearly with the distance x along the plate, is suddenly increased from that of the ambient fluid and the impulsive motion of the free stream velocity. Transient numerical solutions of the governing equations have been presented over the range of physically relevant buoyancy parameter values, $\lambda > -1$, which model the cases in which the buoyancy forces are both assisting and opposing the free stream.

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